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THE MAGIC DIRECTION

T. C. ROY

Physics Department, Jadavpur University, Calcutta.

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Adair (1961) and Shafer *et al* (1963) recently made use of the magic direction for the convenience of analysing the polarization data. It is our purpose to show a few interesting properties of the parity changing term and their effects on this magic direction. The decay matrix element for a half spin particle with momentum $\vec{p}$ is proportional to 1 or $\vec{\sigma}\cdot\vec{p}$ without or with parity change. If $\vec{n}$ is any direction, the polarization along $\vec{n}$ is $<\psi\vec{\sigma}\cdot\vec{n}\psi>$. If however $\vec{\sigma}\cdot\vec{p}$ term is present it changes the l of the state keeping j and jz unaltered. To prove this we note,

$$[\vec{\sigma}\cdot\vec{p},\ L_z+\tfrac{1}{2}\sigma_z]_-=0\quad\{\hbar=1,\text{etc}\}\qquad\ldots\quad(1)$$

so that jz is not tampered. Also since,

$$\{\vec{\sigma}\cdot\vec{p},,\ \vec{\sigma}\cdot\vec{L}+1\}_+=0\qquad\ldots\quad(2)$$

it follows,

$$[\vec{\sigma}\cdot\vec{p},\ (\vec{\sigma}\cdot L+1)^2]=0\qquad\ldots\quad(3)$$

i.e. $(\vec{\sigma}\cdot\vec{L}+1)^2=(\vec{L}+\tfrac{1}{2}\vec{\sigma})^2+1/4$ has the same eigen value $(j+\frac{1}{2})^2$ for the new state. Further from (3) and (2),

$$(j+\tfrac{1}{2})^2\vec{\sigma}\cdot\vec{p}\psi=(L^2+\sigma\cdot L+1)\vec{\sigma}\cdot\vec{p}\psi=L^2\vec{\sigma}\cdot\vec{p}\psi-(\vec{\sigma}\cdot\vec{p})(\vec{\sigma}\cdot\vec{L}+1)\psi\qquad(4)$$

But $\quad(j+\frac{1}{2})^2\psi=L^2\psi(\vec{\sigma}\cdot\vec{L}+1)\psi=l(l+1)\psi+(\vec{\sigma}-\vec{L}+1)\psi \qquad\ldots\quad(5)$

Hence (4) becomes,

$$(j+\tfrac{1}{2})^2\vec{\sigma}\cdot\vec{p}\psi=L^2\vec{\sigma}\cdot\vec{p}\psi-\vec{\sigma}\cdot\vec{p}[(j+\tfrac{1}{2})^2-l(l+1)]\psi\qquad\ldots\quad(6)$$

or, $\quad\vec{L^2}\vec{\sigma}\cdot\vec{p}\psi=[2(j+{}^{m})^2-l(l+1)]\vec{\sigma}\cdot\vec{p}\psi$

i.e. $(\vec{\sigma}\cdot\vec{p}\psi)$ has l value either $(l+1)$ or $(l-1)$ according as $j=l-s$ or $l+s$. We thus prove that under $\vec{\sigma}\cdot\vec{p}$

$$|l,j,jz>\Longleftrightarrow l\,|\pm 1,j,jz>$$

or in particular,

$$P^{\pm 3/2}_{3/2} \Longleftrightarrow D^{3/2}_{3/2} \quad \text{etc.}$$

Now the magic direction, $\vec{m}$ is such that $\vec{p}$ bisects the angle between $\vec{m}$ and $\vec{n}$ i.e.

$$\vec{m} = -n + 2(\vec{n} \cdot \vec{p})\vec{p}$$

The polarisation along $\vec{m}$ for $(\vec{\sigma} \cdot \vec{p}\psi)$ is,

$$< \psi\vec{\sigma} \cdot \vec{p}\ \vec{\sigma} \cdot \vec{m}\ \vec{\sigma} \cdot \vec{p}\psi >$$

$$= < \psi\vec{\sigma} \cdot \vec{p}\ \vec{\sigma} \cdot [-\vec{n} + 2(\vec{n} \cdot \overleftarrow{p})\overleftarrow{p}]\ \overleftarrow{\sigma} \cdot \overleftarrow{p}\psi$$

$$= < \psi\overleftarrow{\sigma} \cdot \overleftarrow{n}\psi$$

which shows that the percentage of polarization along magic direction m for $|l+1, j, jz>$ when it is 100% polarized along n is the same as the percentage of polarization along n for $|l, j, jz>$ when it is 100% polarized along m and vice versa. This result is nicely used by Shafer *et al.*

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